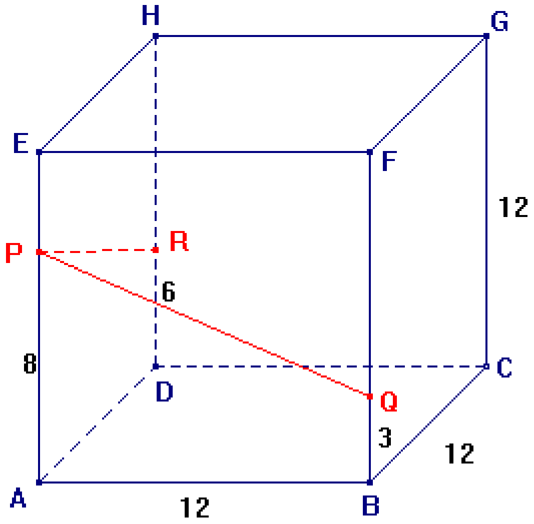
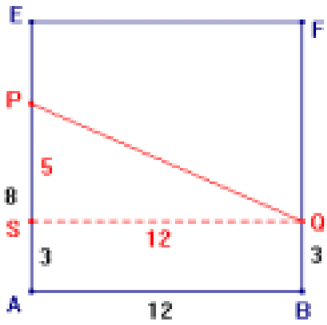
Afstanden en hoeken in de ruimte
Oplossing oefening 2



Geg.: Kubus $\begin{pmatrix} EFGH \\ ABCD \end{pmatrix}$

$|AB| = 12$
$P \in [AE]$, $|PA| = 8$
$Q \in [BF]$, $|BQ| = 3$
$R \in [DH]$, $|RD| = 6$

Gevr.: $Q\hat{P}R$



- Berekening |PQ| met stelling van Pythagoras in voorvlak van de kubus (bepaal S op [AE] zodat QS // AB)

  $|PQ|^2 = |PS|^2 + |QS|^2 \quad \Rightarrow |PQ|^2 = 5^2 + 12^2 = 25 + 144 = 169$
  $\Rightarrow |PQ| = \sqrt{169} = 13$

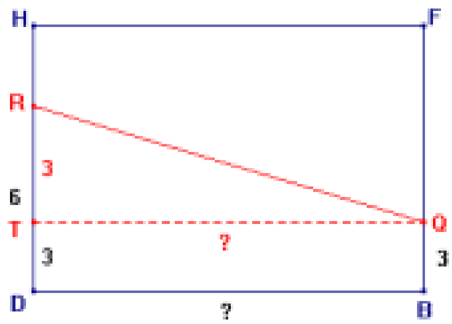
- Analoog: berekening |PR| met stelling van Pythagoras in vl(ADEH)

  $|PR|^2 = 12^2 + 2^2 = 148 \quad \Rightarrow |PR| = \sqrt{148}$

- In vl(BDFH) kunnen we |QR| berekenen.
  Hiertoe moeten we eerst |BD| bepalen (diagonaal van het grondvlak).

  $|BD|^2 = |AB|^2 + |AD|^2 \quad \Rightarrow |BD|^2 = 12^2 + 12^2 = 144 + 144 = 288$
  $\Rightarrow |BD| = \sqrt{288}$

  $|QR|^2 = |QT|^2 + |RT|^2 \quad \Rightarrow |QR|^2 = (\sqrt{288})^2 + 3^2 = 288 + 9 = 297$
  $\Rightarrow |QR| = \sqrt{297}$



- Nu kunnen we $Q\hat{P}R$ berekenen met de cosinusregel in ΔPQR:

$$|QR|^2 = |PQ|^2 + |PR|^2 - 2.|PQ|.|PR|.\cos(Q\hat{P}R) \quad \Rightarrow \cos(Q\hat{P}R) = \frac{|QR|^2 - |PQ|^2 - |PR|^2}{-2.|PQ|.|PR|}$$

$$\text{Dus } \cos(Q\hat{P}R) = \frac{297 - 169 - 148}{-2.13.\sqrt{148}} = 0{,}06323 \quad \Rightarrow Q\hat{P}R = 86°\ 22'\ 29''$$